

Polarization of Thermal X-rays from Isolated Neutron Stars

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ABSTRACT

Since the opacity of a magnetized plasma depends on polarization of radiation, the radiation emergent from atmospheres of neutron stars with strong magnetic fields is expected to be strongly polarized. The degree of linear polarization, typically $\sim 10 - 30\%$, depends on photon energy, effective temperature and magnetic field. The spectrum of polarization is more sensitive to the magnetic field than the spectrum of intensity. Both the degree of polarization and the position angle vary with the neutron star rotation period so that the shape of polarization pulse profiles depends on the orientation of the rotational and magnetic axes. Moreover, as the polarization is substantially modified by the general relativistic effects, observations of polarization of X-ray radiation from isolated neutron stars provide a new method for evaluating the mass-to-radius ratio of these objects, which is particularly important for elucidating the properties of the superdense matter in the neutron star interiors.

Subject headings: polarization — pulsars: general — stars: magnetic fields — stars: neutron — X-rays: stars

1. Introduction

Optical and radio polarimetry has proven to be a powerful tool to elucidate properties of various astrophysical objects. For instance, virtually all our knowledge about the orientations of the magnetic and rotation axes of radio pulsars comes from analyzing the swing of polarization position angle within the pulse (see, e.g., Manchester & Taylor 1977; Lyne & Manchester 1988). On the other hand, X-ray polarimetry has remained an underdeveloped field of astrophysics. Although various X-ray polarimeters have been designed (e.g., Kaaret et al. 1990; Weisskopf et al. 1994; Elsner et al. 1997; Marshall et al. 1998), and importance of X-ray polarimetry convincingly

demonstrated (Mészáros et al. 1988), most recent measurements of X-ray polarization has been made as long ago as in 1977, with the *OSO 8* mission (Weisskopf et al. 1978). Nevertheless, it is expected that X-ray polarimeters will be launched in near future (see, e.g., Tomsick et al. 1997). To develop efficient observational programs for forthcoming X-ray missions whose objectives will include X-ray polarimetry, the problem of polarization of various X-ray sources should be carefully analyzed, with the main emphasis on new astrophysical information to be inferred from such observations.

In the present paper we consider polarization of thermal X-ray radiation from isolated neutron stars (NSs) with strong magnetic fields. Recent observations with the *ROSAT* and *ASCA* missions have shown that several such objects are sufficiently bright for polarimetric observations — e.g., the radio pulsars PSR 0833–45 and PSR 0656+14 (see Ögelman 1995, and Becker & Trümper 1997, for reviews), and the radio-quiet NSs RX J0822–4300 (Zavlin, Trümper, & Pavlov 1999) and RX J1856.5–3754 (Walter, Wolk, & Neuhauser 1996). Their soft X-ray radiation was interpreted as emitted from NS surface layers (atmospheres) with effective temperatures T_{eff} in the range of $(0.3 - 3) \times 10^6$ K. Since photons of different energies escape from different depths of the NS atmosphere with temperature growing inward, the spectrum of the thermal radiation may substantially deviate from the blackbody spectrum (Pavlov & Shibano 1978). Moreover, if there is a strong magnetic field in the NS atmosphere, such that the electron cyclotron energy $E_{Be} = \hbar e B / m_e c = 11.6 (B / 10^{12} \text{ G}) \text{ keV}$ is comparable to or exceeds the photon energy E , then the radiation propagates as two normal modes (NMs) with different (approximately orthogonal) polarizations and opacities (Gnedin & Pavlov 1974). For typical magnetic fields, $B \sim 10^{11} - 10^{13} \text{ G}$, the NMs at soft X-ray energies, $E \ll E_{Be}$, are linearly polarized in a broad range of wavevector directions, and the opacity κ_e of the so-called extraordinary mode (polarized perpendicular to \mathbf{B}) is much smaller than that of the ordinary mode, $\kappa_e \sim (E/E_{Be})^2 \kappa_o$. As a result, the extraordinary mode escapes from deeper and hotter layers, so that the emergent radiation acquires strong linear polarization perpendicular to the local magnetic field (Pavlov & Shibano 1978). Polarization of the observed radiation depends on the distribution of magnetic field and temperature over the visible NS surface. If these distributions are axisymmetric, the polarization is a function of the angle Θ between the symmetry (magnetic) axis and the line of sight. If the direction of the magnetic axis varies due to NS rotation, the polarization patterns show pulsations with the period of rotation, so that measuring the polarization pulse profile allows one to constrain the orientations of the axes. Due to the gravitational bending of the photon trajectories, the visible fraction of the NS surface grows with increasing the NS mass-to-radius ratio, M/R , which reduces the net polarization because the observer sees additional regions with differently directed magnetic fields. On the other hand, the gravitational field affects the magnetic field geometry making the field more tangential (Ginzburg & Ozernoy 1965), which increases the observed polarization. These GR effects enable one, in principle, to constrain M/R by measuring the X-ray polarization. We demonstrate that the expected X-ray polarization of the thermal NS radiation is high enough to be measured with soft-X-ray polarimeters in a modest exposure time, and these measurements can provide important new information on both the geometry of the magnetic field and the

mass-to-radius ratio.

2. Description of calculations

The intensity I and the Stokes parameters Q and U at a given point of the NS surface can be expressed as (Gnedin & Pavlov 1974)

$$I = I_o + I_e , \quad (1)$$

$$Q = (I_o - I_e) p_L \cos 2\chi_o , \quad (2)$$

$$U = (I_o - I_e) p_L \sin 2\chi_o , \quad (3)$$

where I_o and I_e are the intensities of the ordinary and extraordinary modes, $p_L = (1 - \mathcal{P}^2)/(1 + \mathcal{P}^2)$ is the degree of linear polarization of the NMs (\mathcal{P} is the ellipticity, i.e., the ratio of the minor axis to the major axis of the polarization ellipse), and χ_o is the angle between the major axis of the polarization ellipse of the ordinary mode and the x axis of a reference frame in which the Stokes parameters are defined.

We calculate the local NM intensities with the aid of NS atmosphere models (e.g., Pavlov et al. 1994, 1995). In the present work we assume that the surface temperature is high enough for the atmospheric matter to be completely ionized. If the NS surface is covered with a hydrogen atmosphere, this assumption is justified at $T_{\text{eff}} \gtrsim 10^6$ K, for typical magnetic fields of NS (Shibanov et al. 1993). The local intensities I_o and I_e depend on the photon energy, magnetic field, and direction of emission.

In the dipole approximation, valid at photon and particle energies much lower than $m_e c^2$, the degree of linear polarization of NMs can be expressed as

$$p_L = \frac{|q| \sin^2 \theta'}{\sqrt{4 \cos^2 \theta' + q^2 \sin^4 \theta'}} \quad (4)$$

where θ' is the angle between the magnetic field \mathbf{B} and the unit wavevector $\hat{\mathbf{k}}'$ at the NS surface. The (angle-independent) parameter q is determined by the components of the Hermitian part of the polarizability tensor in the coordinate frame with the polar axis along the magnetic field (Gnedin & Pavlov 1974). The parameter q depends on photon energy and magnetic field (e.g., Pavlov, Shibanov, & Yakovlev 1980; Bulik & Pavlov 1996). For instance, if the hydrogen plasma is completely ionized, and the electron-positron vacuum polarization by the magnetic field can be neglected, this parameter equals

$$q = \frac{E'^2(E_{Be}^2 + E_{Bi}^2 - E_{Be}E_{Bi}) - E_{Be}^2E_{Bi}^2}{E'^3(E_{Be} - E_{Bi})} , \quad (5)$$

where E' is the photon energy as measured at the NS surface, $E_{Bi} = (m_e/m_p)E_{Be} = 6.32 (B/10^{12} \text{ G}) \text{ eV}$ is the ion (proton) cyclotron energy. If the photon energy is much greater

than the ion cyclotron energy, the q parameter is particularly simple: $q = E_{Be}/E'$. This means that the NMs are linearly polarized, $p_L \simeq 1$, in a broad range of directions, $\sin^2 \theta' \gg 2E'/E_{Be}$, at photon energies much lower than the electron cyclotron energy. It should be mentioned that equations (1)–(4) imply that the NM polarizations are orthogonal to each other (in particular, $\chi_e = \chi_o \pm \pi/2$). This condition is fulfilled in a broad domain of photon energies and directions, except for a few special values of θ', E' (e.g., Pavlov & Shibano 1979; Bulik & Pavlov 1996). Within the same approximations, the angle χ_o coincides with the azimuthal angle of the magnetic field in a reference frame whose polar axis is parallel to $\hat{\mathbf{k}}'$.

To find the observed flux F_I and the observed Stokes parameters F_Q and F_U , one should sum contributions from all the elements of the visible NS surface. We assume the magnetic field and the temperature distribution are axially symmetric and define F_Q and F_U in the reference frame such that the axis of symmetry \mathbf{m} lies in the xz plane, the z axis is directed along the line of sight. In such a frame $F_U = 0$, and F_I, F_Q are functions of the angle Θ between \mathbf{m} and $\hat{\mathbf{z}}$. The ratio $P_L = -F_Q/F_I$ gives the observed degree of linear polarization, $|P_L|$, and the observed position angle: the polarization direction is perpendicular or parallel to the projection of $\hat{\mathbf{m}}$ onto the sky plane for $F_Q > 0$ or $F_Q < 0$, respectively.

Since the NS radius R is comparable with the gravitational (Schwarzschild) radius $R_g = 2GM/c^2$, the photon energy and the wavevector and polarization directions change in the course of propagation in the strong gravitational field. We will assume that the NS gravitational field is described by the exterior Schwarzschild solution. Since strong magnetic fields ($B \gg 10^{10}$ G) are needed to obtain measurable polarization in the X-ray range, and all observed NSs with strong magnetic fields are not very fast rotators ($P > 10$ ms), the effects of rotation on the metric and on the observed radiation are very small. For the Schwarzschild metric, the observed energy is redshifted as $E = g_r E'$, where $g_r = (1 - R_g/R)^{1/2}$ is the redshift factor. The observed wavevector is inclined to the emitted wavevector by the angle $K - \vartheta$, $\hat{\mathbf{k}}' \cdot \hat{\mathbf{z}} = \cos(K - \vartheta)$, where K is the colatitude of the emitting point in the reference frame with the origin at the NS center and the z axis directed towards the observer, ϑ is the angle between the normal to the surface and the wavevector direction $\hat{\mathbf{k}}'$ at the emitting point. The angle ϑ would coincide with K in flat space-time. In the Schwarzschild geometry, K always exceeds ϑ , i.e., some part of the NS back hemisphere is visible. For instance,

$$K = a \int_0^{R_g/R} \frac{dx}{\sqrt{1 - a^2(1 - x)^2}}, \quad (6)$$

for $K \leq \pi$ ($g_r \geq 0.65$), where $a = R \sin \vartheta / (R_g g_r)$ is the impact parameter in units of R_g (see, e.g., Zavlin, Shibano, & Pavlov 1995). In particular,

$$K - \vartheta \simeq u \tan \frac{\vartheta}{2} + \frac{1}{16} u^2 \left[\frac{15(\vartheta - \sin \vartheta)}{\sin^2 \vartheta} + 7 \tan \frac{\vartheta}{2} \right] \quad (7)$$

for $u \equiv R_g/R \ll 1$.

The bending of the photon trajectories is associated with changing the direction of linear polarization. The polarization direction rotates in such a way that it keeps fixed orientation with respect to the normal to the trajectory plane (e.g., Pineault 1977), remaining perpendicular to the wavevector. Without this rotation, the angles χ_o and χ_e would be conserved: $\chi_o^{\text{obs}} = \phi$ at the observation point, where $\phi = \tan^{-1}(B_y/B_x)$ is the azimuthal angle of the magnetic field at the emitting point in the x, y, z frame. To find χ_o^{obs} with allowance for the GR effects, it is convenient to introduce the frame x', y', z' such that the z' axis is parallel to $\hat{\mathbf{k}}'$, and the photon trajectory is in the $x'z'$ plane (see Fig. 1). The unit vectors along the axes of the two frames are connected with each other as follows

$$\hat{\mathbf{x}}' = \cos \varphi \cos(K - \vartheta) \hat{\mathbf{x}} + \sin \varphi \cos(K - \vartheta) \hat{\mathbf{y}} - \sin(K - \vartheta) \hat{\mathbf{z}}, \quad (8)$$

$$\hat{\mathbf{y}}' = -\sin \varphi \hat{\mathbf{x}} + \cos \varphi \hat{\mathbf{y}}, \quad (9)$$

$$\hat{\mathbf{z}}' = \cos \varphi \sin(K - \vartheta) \hat{\mathbf{x}} + \sin \varphi \sin(K - \vartheta) \hat{\mathbf{y}} + \cos(K - \vartheta) \hat{\mathbf{z}}, \quad (10)$$

where φ is the azimuthal angle of the emitting point in the x, y, z frame. Using the conditions that the angle between $\hat{\mathbf{y}}'$ and the polarization direction is conserved, and the polarization direction is perpendicular to $\hat{\mathbf{z}}$ at the observation point, we obtain $\chi_o^{\text{obs}} = \varphi + \phi'$, where ϕ' is the azimuthal angle of the magnetic field in the x', y', z' frame: $\phi' = \tan^{-1}(B_{y'}/B_{x'})$. The angle ϕ' depends on $K - \vartheta$ and φ , and it tends to $\phi - \varphi$ when $R_g/R \rightarrow 0$.

With allowance for the above-described gravitational effects, the observed flux F_I and the Stokes parameter F_Q are given by the following integrals over the visible NS surface (see Zavlin et al. 1995):

$$F_I(E, \Theta) = \frac{R^2}{d^2} g_r \int_0^1 \mu d\mu \int_0^{2\pi} d\varphi (I_o + I_e) = F_o(E, \Theta) + F_e(E, \Theta), \quad (11)$$

$$F_Q(E, \Theta) = \frac{R^2}{d^2} g_r \int_0^1 \mu d\mu \int_0^{2\pi} d\varphi (I_o - I_e) p_L \cos 2(\varphi + \phi'), \quad (12)$$

where d is the distance, $\mu = \cos \vartheta$, and the integrands are taken at the photon energy $E' = E/g_r$.

To calculate the integrands, we should know the magnetic field at the NS surface as a function of μ and φ . We consider a dipole magnetic field in the Schwarzschild metric. According to Ginzburg & Ozernoy (1965), the field equals

$$\mathbf{B} = B_p \frac{(2 + f)(\hat{\mathbf{r}} \cdot \hat{\mathbf{m}})\hat{\mathbf{r}} - f\hat{\mathbf{m}}}{2}, \quad (13)$$

where B_p is the field strength at the magnetic pole, $\hat{\mathbf{r}}$ is the unit radius vector of a surface point, and $\hat{\mathbf{m}}$ is the unit vector of the magnetic moment. The parameter

$$f = 2 \frac{u^2 - 2u - 2(1 - u) \ln(1 - u)}{[u^2 + 2u + 2 \ln(1 - u)] \sqrt{1 - u}} \quad (14)$$

accounts for the GR effect. For $u \ll 1$ ($R \gg R_g$), we have $f(u) \simeq 1 + u/4 + 11u^2/80$. The radial and tangential components of the magnetic field are $B_r = B_p \cos \gamma$ and $B_t = (B_p/2)f \sin \gamma$, where

$\cos \gamma = \hat{\mathbf{r}} \cdot \hat{\mathbf{m}} = \sin \Theta \sin K \cos \varphi + \cos \Theta \cos K$. Since $f > 1$, the GR effect makes the magnetic field more tangential. The projections of \mathbf{B} onto the x, y, z and x', y', z' axes can be easily found with the aid of equations

$$\hat{\mathbf{r}} = \sin K \cos \varphi \hat{\mathbf{x}} + \sin K \sin \varphi \hat{\mathbf{y}} + \cos K \hat{\mathbf{z}}, \quad (15)$$

$$\hat{\mathbf{m}} = \sin \Theta \hat{\mathbf{x}} + \cos \Theta \hat{\mathbf{z}}, \quad (16)$$

and equations (8)–(10). The strength of the magnetic field is

$$B = (B_p/2) \left[(4 - f^2) \cos^2 \gamma + f^2 \right]^{1/2} = B_p f \left[4 - (4 - f^2) \cos^2 \theta_B \right]^{-1/2}, \quad (17)$$

where θ_B is the angle between \mathbf{B} and the normal to the surface $\hat{\mathbf{r}}$.

The integration over the NS surface (eqs. [11], [12]) proceeds as follows. For each point of the μ, φ grid, we calculate the colatitude $K(\vartheta)$ from equation (6) and the components of the radius vector $\hat{\mathbf{r}}$ in the x, y, z and x', y', z' frames (eqs. [15] and [8]–[10]). This gives us the projections and strength of the local magnetic field \mathbf{B} (eqs. [13] and [17]), the angles $\phi' = \tan^{-1}(B_{y'}/B_{x'})$, $\theta' = \cos^{-1}(B_{z'}/B)$, and $\theta_B = \cos^{-1}(\mathbf{B} \cdot \hat{\mathbf{r}})$, and the degree of NM polarization p_L (eq. [4]). To obtain the intensities of the extraordinary and ordinary modes of radiation emitted to the observer from a given surface point, one needs to know the local depth dependences of the temperature and density in the NS atmosphere, determined by the local values of B , θ_B , and T_{eff} . These dependences are obtained by interpolation within a set of the diffusion atmosphere models (Pavlov et al. 1995). Then, the I_o and I_e intensities are computed as described by Pavlov et al. (1994) and Shibano & Zavlin (1995). Subsequent numerical integration over μ, φ gives us F_I , F_Q , the degree of the observed linear polarization and the position angle.

3. Results

To demonstrate how the observed linear polarization depends on photon energy, magnetic field, and mass-to-radius ratio, we consider a NS covered with a hydrogen atmosphere with a uniform effective temperature and a dipole magnetic field. We present the results for $T_{\text{eff}} = 1 \times 10^6$ K, $B_p/(10^{12} \text{ G}) = 0.3, 1.0, 3.0$ and 10.0 . We choose a standard NS radius $R = 10$ km and three NS masses, $M/M_\odot = 0.66, 1.40$ and 1.92 , from an allowed domain in the M – R diagram (filled circles in Fig. 2). These masses correspond to the redshift parameters $g_r = 0.90, 0.77$ and 0.66 , and the surface gravitational accelerations $g/(10^{14} \text{ cm}^2 \text{ s}^{-1}) = 0.97, 2.43$, and 3.89 . It should be noted that the properties of the emitted radiation are almost independent of the g value, so that the gravitational effects on the observed radiation are determined mainly by the redshift parameter g_r , i.e., by the mass-to-radius ratio.

The left panel of Figure 3 demonstrates the observed photon spectral fluxes F_I (eq. [11]) from a NS with the magnetic field $B_p = 1 \times 10^{12}$ G at the magnetic pole and the magnetic axis perpendicular to the line of sight, $\Theta = 90^\circ$. The flux is normalized to a distance of 1 kpc. The

spectra are presented for the three values of the redshift parameter g_r . To demonstrate the effect of the interstellar absorption, we plot the spectra for the effective hydrogen column densities $n_H = 0$ (unabsorbed flux), 1×10^{20} and $1 \times 10^{21} \text{ cm}^{-2}$ (the latter two are shown for $g_r = 0.77$ only). The effect of redshift is clearly seen, as well as that of the interstellar absorption: spectral maxima shift from 0.15 keV at $n_H = 0$ to 0.6 keV at $n_H = 1 \times 10^{21} \text{ cm}^{-2}$. The contributions from the extraordinary and ordinary modes (fluxes F_e and F_o) to the unabsorbed spectrum $F_I = F_e + F_o$ are shown in the right panel of Figure 3 for $g_r = 0.77$. At energies around the maxima of the flux spectra the radiative opacity of the ordinary mode significantly exceeds that of the extraordinary mode. Hence, the extraordinary mode is emitted from deeper and hotter atmosphere layers, providing the main contribution to the total flux (see Pavlov et al. 1995 for details). At higher energies ($E' \gtrsim 3^{-1/2} E_{Be}$) the relation between the two opacities, and the two NM fluxes, is reversed (Kaminker, Pavlov, & Shibano 1982). This leads to changing the sign of F_Q (i.e., to the jump of the polarization position angle by $\pi/2$).

Several examples of the dependences of $P_L = -F_Q/F_I$ on photon energy are presented in Figure 4, for $B_p = 1 \times 10^{12} \text{ G}$ and different values of the angle Θ and the redshift parameter g_r . In the soft X-ray range, where the thermal NS radiation is most easily observed, P_L is positive, i.e., the polarization direction is perpendicular to the projection of the NS magnetic axis onto the image plane. In this energy range the ordinary mode is emitted from superficial layers with lower temperature, whereas the extraordinary mode is formed in deeper and hotter layers with a larger temperature gradient. As a result, the ratio F_e/F_o grows with E at low energies until this effect is compensated by the decrease of the difference between the extraordinary and ordinary opacities (see the right panel of Fig. 3). At higher energies the extraordinary and ordinary fluxes approach each other, so that F_e/F_o decreases with increasing E reaching unity at $E \approx 0.3g_r E_{Be}$ (hereafter, E_{Be} and E_{Bi} are the cyclotron energies for the magnetic field B_p). Since $p_L \approx 1$ at $E_{Bi} \ll E' \ll E_{Be}$, and $\varphi + \phi'$ does not depend on E' , it follows from equations (11) and (12) that $P_L \propto (F_e - F_o)/(F_e + F_o)$, with a proportionality coefficient independent of E . This explains the energy dependence of P_L in Figure 4. In particular, starting from energies $E \sim g_r E_{Bi}$, P_L grows with E until it reaches a maximum (at $E \sim 1 \text{ keV}$ for $B_p = 1 \times 10^{12} \text{ G}$). At higher energies the polarization spectra steeply decrease with increasing E , reach zero at $E \sim 0.3g_r E_{Be}$, where the contributions from the two NMs cancel each other, and become negative (polarization direction becomes parallel to the \mathbf{m} projection) at higher energies, where the flux decreases exponentially at the effective temperature chosen.

As expected, the degree of polarization grows with increasing Θ from 0° to 90° . It equals zero at $\Theta = 0^\circ$ (the magnetic axis is parallel to the line of sight) because the contributions from the azimuthal angles φ and $\varphi + \pi/2$ to F_Q (eq. [12]) are polarized in orthogonal directions, being of the same magnitude ($B_{y'} = 0$, $\phi' = 0$, $\cos 2\varphi = -\cos 2(\varphi \pm \pi/2)$). The polarization is maximal at $\Theta = 90^\circ$, when the magnetic lines are seen by the observer almost parallel to the magnetic axis on a substantial (central) part of the visible stellar disk.

We see from Figure 4 that the degree of polarization decreases with increasing M/R (or

decreasing g_r). The main reason is that the observer sees a larger fraction of the whole NS surface due to stronger bending of photon trajectories. As a result, the overall pattern of the magnetic lines on the visible NS disk becomes more nonuniform, which leads to additional “cancellation” of the mutually orthogonal polarizations emitted from parts of the surface with orthogonally oriented magnetic line projections. This effect is partly compensated by the other GR effect, more tangential dipole magnetic field in the Schwarzschild metric (the parameter f in eq. [13] equals 1.08, 1.14, and 1.22 for $g_r = 0.90$, 0.77, and 0.66, respectively), but the effect of bending prevails.

Figure 5 demonstrates the effect of magnetic field strength on the degree of polarization. In particular, P_L in the soft X-ray range grows with B_p for typical NS magnetic fields. When the magnetic field is relatively small, $B_p \lesssim 1 \times 10^{12}$ G, the fast growth of P_L is due to the increasing difference between the extraordinary and ordinary opacities (hence, increasing differences between the escape depths and between the NM intensities). Slower growth of P_L at intermediate fields (around $B_p \sim 1 \times 10^{12}$ G) is mainly due to the decrease of the surface temperature with increasing B (Pavlov et al. 1995), which reduces the ordinary flux F_o and the ratio F_o/F_e . With further increase of magnetic field, the surface temperature ceases to decrease, and $P_L(B_p)$ saturates in the soft X-ray range at $B_p \gtrsim 3 \times 10^{12}$ G, until the field becomes so strong, $B_p \gtrsim 3 \times 10^{13}$ G, that $g_r E_{Bi}$ becomes comparable with E , and the proton cyclotron spectral feature gets into the soft X-ray range.

The proton cyclotron feature in the polarization spectrum is shown in Figure 5 for $B_p = 1 \times 10^{13}$ G. The shape of the feature can be explained by the behavior of the parameter q and the NM intensities near the proton cyclotron resonance. According to equation (5), q grows with decreasing photon energy until E' reaches $3^{1/2}E_{Bi}$; then it sharply decreases, crosses zero in the very vicinity of proton cyclotron resonance, at $E' \simeq E_{Bi}(1 + 2m_e/m_p)$, and tends to $-\infty$ ($q \simeq -E_{Be}E_{Bi}^2/E'^3$ at $E' \ll E_{Bi}$). This means that p_L reaches zero, the NM intensities I_e and I_o equal each other, and the integrand of equation (12) equals zero at an energy in the very vicinity of the proton resonance corresponding to the local magnetic field. If the direction of the local magnetic field is such that $\cos 2(\varphi + \phi') > 0$ (see eq. [12]), which roughly corresponds to the projection of the local magnetic field onto the sky plane within $\pm 45^\circ$ of the magnetic axis projection, then the integrand in the expression for $-F_Q$ is positive at energies around the resonance, so that the energy dependence of the integrand for the corresponding surface points looks like an “absorption line” in a positive continuum, with its minimum (zero) value at the local resonance energy. Integration over the area with positive $\cos 2(\varphi + \phi')$ yields a positive contribution to $-F_Q$, with an absorption line somewhat broadened, and its minimum above zero, because of nonuniformity of the magnetic field. On the contrary, the energy dependence of the integrand in the area where $\cos 2(\varphi + \phi') < 0$ looks like an “emission line” on a negative continuum, with its maximum equal zero at the local resonance energy. The integral over this area gives a negative contribution to $-F_Q$, its absolute values are minimal in the energy range which includes the local resonance energies. If the strengths of the local magnetic fields are different in the areas of positive and negative $\cos 2(\varphi + \phi')$, the integration over the whole visible disk results in a complex feature

in the $-F_Q(E)$ and $P_L(E)$ spectra. In Figure 5 this feature is most pronounced at $B_p = 1 \times 10^{13}$ G and $\Theta = 90^\circ$. It consists of two components: the absorption component with a sharp minimum at $E \simeq 28$ eV corresponding to the field at the magnetic equator ($B_e = B_p f/2$), and the emission component at energies below $E \simeq 48$ eV, corresponding to the field at the magnetic pole. The feature is also clearly seen at the same B_p and $\Theta = 45^\circ$, whereas only the emission component of the feature is seen at $E > 10$ eV in the curves of $P_L(E)$ for $B_p = 3 \times 10^{12}$ G. If the magnetic field is superstrong, $B \sim 10^{14} - 10^{15}$ G, as suggested for anomalous X-ray pulsars and soft gamma repeaters, the proton cyclotron feature gets into the soft or medium X-ray range, and its detection would enable one to measure directly the magnetic field strength.

Generally, the polarization spectra are much more sensitive to the strength of magnetic field than the intensity spectra (Fig. 6). The main effect of B on $F_I(E)$ spectra in the soft energy range is a shallow proton cyclotron absorption feature (see the spectrum for $B_p = 1 \times 10^{13}$ G in Fig. 6). At $E \sim 0.2 - 1.0$ keV the intensity spectra at different magnetic fields typical for NSs are almost indistinguishable, with the main contribution coming from the extraordinary mode whose spectrum is almost independent of B at $E_{Bi} \ll E' \ll E_{Be}$.

If the angle α between the magnetic and rotation axes differs from zero, the projection of \mathbf{m} onto the sky changes its orientation with the period of rotation. This means that the angle Θ and the polarization position angle δ with respect to a fixed (nonrotating) direction also oscillate with the rotation period P :

$$\cos \Theta = \cos \zeta \cos \alpha + \sin \zeta \sin \alpha \cos 2\pi\Phi, \quad (18)$$

$$\tan \delta = \frac{\cos \alpha \sin \zeta - \sin \alpha \cos \zeta \cos 2\pi\Phi}{\sin \alpha \sin 2\pi\Phi}, \quad (19)$$

where ζ is the angle between $\mathbf{\Omega}$ (NS rotation axis) and the line of sight, $\Phi = t/P$ is the rotation phase, the angle δ is counted from the projection of $\mathbf{\Omega}$ onto the sky. Equation (19) is applicable in the case of polarization perpendicular to the projection of \mathbf{m} onto the sky plane ($P_L > 0$); for $P_L < 0$, the left-hand-side is replaced by $\cot \delta$.

Figure 7 shows several characteristic dependences of $P_L(\Phi)$ and $\delta(\Phi)$ for $E = 0.3$ keV, $g_r = 0.77$, and a few sets of ζ, α . In the particular case of an orthogonal rotator, $\zeta = \alpha = 90^\circ$, we have $\Theta = \Phi$, $\delta = 0$, i.e., the degree of polarization oscillates between zero (at $\Phi = 0, 0.5$) and a maximum value, $P_L = 25\%$ at $\Phi = 0.25, 0.75$, showing two pulses per period, while the position angle remains constant (the polarization is oriented along the direction of the rotation axis). For the case $\zeta = 60^\circ$, $\alpha = 50^\circ$, the minimum polarization, $P_L = 1\%$ at $\Phi = 0, 1$, corresponds to $\Theta = \zeta - \alpha = 10^\circ$. The polarization pulse has two maxima per period, $P_L = 25\%$ at $\Phi = 0.36, 0.64$ (corresponding to $\Theta = 90^\circ$) and a local minimum, $P_L = 22\%$ at $\Phi = 0.5$ (corresponding to $\Theta = \zeta + \alpha = 110^\circ$). The position angle oscillates around $\pi/2$ (or $-\pi/2$). For $\zeta = \alpha = 45^\circ$, P_L has one broad maximum per period, of the same height as for the orthogonal rotator, when $\Theta = 90^\circ$ at $\Phi = 0.5$. The position angle swings from 0 to π during the period, crossing $\pi/2$ at the phases of maximum polarization. Finally, for $\zeta = 40^\circ$, $\alpha = 10^\circ$, the phase dependence of the degree of polarization is almost sinusoidal, with one maximum per period; it oscillates between

7% and 16%. The position angle oscillates around $\pi/2$ (or $-\pi/2$) with a small amplitude because of the small value of α . Thus, we see that a variety of the phase dependences of P_L and δ can be obtained for various combinations of ζ, α , which is potentially useful for evaluating these angles from polarimetric observations.

4. Discussion

As we see from the examples above, the degree of linear polarization of thermal NS radiation is quite high, up to 20%–50% in pulse peaks, and about twice lower for the phase-averaged polarization, for typical NS parameters. Optimal energies for observing the polarization are in the soft X-ray range, ~ 0.1 –1 keV, for typical surface temperatures of young and middle-aged NSs. The polarization can be observed at these energies with the use of multilayer coated mirrors which provide high reflectivity at large grazing angles — see Marshall et al. (1998) for a concept for a satellite-borne polarimeter (PLEXAS) which would be able to measure the linear polarization from brightest thermally emitting pulsars to an accuracy of 1%–3% during modest exposure times ~ 30 –100 ks. The polarization can be measured in several narrow energy bands. Complementing the spectral flux with the polarization spectral data and comparing the both with the NS atmosphere models would considerably narrow allowed ranges for NS parameters such as the magnetic field and effective temperature. Particularly strong constraints could be obtained if the X-ray polarimetric measurements are supplemented by measuring optical-UV polarization of the same sources. Several middle-age pulsars have been observed successfully in the optical-UV range with the *Hubble Space Telescope* (e.g., Pavlov, Welty, & Córdoba 1997, and references therein). Although the expected polarization in this range is lower than in soft X-rays, it still can be as high as 5%–15% (see Figures 4 and 5), so that UV-optical polarimetric observations of these sources seem quite feasible.

Using detectors with high timing resolution (e.g., microchannel-based photon counters) for polarimetric observations of pulsars would allow one to measure phase dependences of the degree of polarization and the position angle. These observations would be most useful to determine the inclinations of the rotation and magnetic axis (ζ and α). Although radio band polarization data have been widely used to constrain these angles, the results are often ambiguous because the same behavior of the position angle can be fitted with different combinations of ζ and α . An additional difficulty in interpreting the radio polarization data is caused by shortness of the duty cycle of radio pulsars — the radio flux is too low during a substantial fraction of the period to measure the polarization. Since the X-ray flux remains bright during the whole period, measuring the position angle and the degree of polarization in X-rays would enable one to infer the orientation of the pulsar axes with much greater certainty.

Of particular interest is the result that the observed polarization is sensitive to the NS mass-to-radius ratio, the most crucial parameter to constrain still poorly known equation of state of the superdense matter in the NS interiors. Since the radio emission of pulsars is generated well

above the NS surface, this ratio cannot be constrained from polarimetric observations of pulsars in the radio band. Although some constraints can be obtained from the pulse profiles of the X-ray flux (Pavlov & Zavlin 1997), the polarization pulse profiles are more sensitive to the gravitational effects.

Primary targets for studying the X-ray polarization of thermal NS radiation are the X-ray brightest, thermally radiating pulsars such as PSR 0833–45 (Vela) and PSR 0656+14. A prototype of another class of promising targets, radio-quiet NSs in supernova remnants, is RX J0822–4300 in Puppis A. X-ray polarimetric measurements would be crucial to establish the strength and geometry of the magnetic field of this putative isolated X-ray pulsar (Pavlov, Zavlin, & Trümper 1999). Also it would be very interesting to study X-ray polarization of X-ray bright, radio-quiet isolated NSs which are not associated with supernova remnants. Particularly interesting is the object RX J1856.5–3754 which has a thermal-like soft X-ray spectrum but does not show pulsations (Walter et al. 1996). The lack of pulsations may be explained either by smallness of the magnetic inclination α , if the magnetic field is typical for NSs ($\sim 10^{11}$ – 10^{13} G), or by a very low surface magnetic field. Polarimetric observations would enable one to distinguish between the two hypotheses — the polarization is expected to be high (and unpulsed) in the former case (unless ζ is also small), and it would be very low if the magnetic field is lower than $\sim 10^{10}$ G. Distinguishing between the two options is needed to choose either low-field or high-field NS atmosphere models — applications of these types of models to interpretation of the multiwavelength observations of this source yield quite different NS parameters (Pavlov et al. 1996).

It follows from our results that thermal radiation from millisecond pulsars, whose typical magnetic fields are $10^8 - 10^9$ G ($E_{Be} \sim 1 - 10$ eV), is not polarized in the X-ray range. However, their polarization is expected to be quite strong in the optical-UV range and can be measured in sufficiently deep observations. The best candidate for such observations is the nearest millisecond pulsar J0437–4715 whose magnetic field, $B \approx 8 \times 10^8$ G, was estimated from radio observations. The thermal radiation from the NS surface, which is expected to be heated up to $\sim 10^5$ K, should prevail over the radiation from the very cool white dwarf companion at $\lambda \lesssim 2000$ Å. This means that polarization from this pulsar could be observed in the far-UV range with the *Hubble Space Telescope*. It should be mentioned that the rapid rotation of millisecond pulsars may affect not only the intensity pulse shape (Braje, Romani, & Rauch 1999), but also the polarization, the effect neglected in the present paper.

In summary, our results demonstrate that including X-ray polarimeters in future X-ray observatories, or launching dedicated X-ray polarimetry missions, would be of great importance for studying NSs. The polarimetric observations would be useful for studying not only the thermal component of the NS X-ray radiation, but also the nonthermal component which dominates in many X-ray emitting pulsars, particularly at higher energies. Further theoretical investigation of polarization of both thermal and nonthermal X-ray emission from NSs are also warranted to provide firm interpretation of future observational data. In particular, it would be useful to consider the polarization with allowance for possible nonuniformity of the temperature distribution

over the NS surface and to study the effects of chemical composition of the NS atmosphere on polarization.

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Fig. 1.— Angles and vectors used in the paper.

Fig. 2.— Mass-radius diagram for NSs. The thick solid curves show the $M(R)$ dependences for several equations of state of the superdense matter (Shapiro & Teukolsky 1983): soft (π), intermediate (FP) and stiff (TI and MF). The thick straight line, $R = 1.5R_g$, corresponds to the most conservative lower limit on NS radius at a given mass. Thin dashed lines correspond to different values of the redshift parameter g_r (the numbers near the lines). The filled circles give the NS mass and radius used in our computations.

Fig. 3.— *Left*: Photon spectral fluxes $F_I(E)$ for $d = 1$ kpc, $B_p = 1 \times 10^{12}$ G, $\Theta = 90^\circ$, and three values of the redshift parameter, $g_r = 0.66$, 0.77 , and 0.90 (dash-dot, solid, and dashed curves, respectively). The numbers near the curves denote the interstellar hydrogen column density n_H , in units of 10^{20} cm^{-2} . *Right*: Contributions from the extraordinary (e) and ordinary (o) NMs to the observed flux for $g_r = 0.77$, $n_H = 0$.

Fig. 4.— Effect of the mass-to-radius ratio and the inclination Θ of the magnetic axis on the polarization spectrum for $B_p = 1 \times 10^{12}$ G. Thin, medium, and thick curves are for $g_r = 0.90$, 0.77 , and 0.66 , respectively.

Fig. 5.— Effect of the strength of magnetic field on the polarization spectrum for $g_r = 0.77$. The curves are plotted for $\Theta = 90^\circ$ (solid) and 45° (dashed), and $B_p/(10^{12} \text{ G}) = 0.3$, 1.0 , 3.0 , and 10.0 (the numbers near the curves).

Fig. 6.— Unabsorbed spectral fluxes $F_I(E)$ for $g_r = 0.77$ and $B_p/10^{12} \text{ G} = 0.3$ (short dashes), 1.0 (solid), 3.0 (long dashes), and 10.0 (dash-dots).

Fig. 7.— Dependences of the degree of polarization and the position angle on NS rotation phase Φ for $g_r = 0.77$, $E = 0.3 \text{ keV}$, and different angles between the rotation axis and the line of sight, and between the magnetic and rotation axes: $\zeta = \alpha = 90^\circ$ (solid), $\zeta = \alpha = 45^\circ$ (long dashes), $\zeta = 60^\circ$ and $\alpha = 50^\circ$ (dot-dashes), and $\zeta = 40^\circ$ and $\alpha = 10^\circ$ (short dashes).













